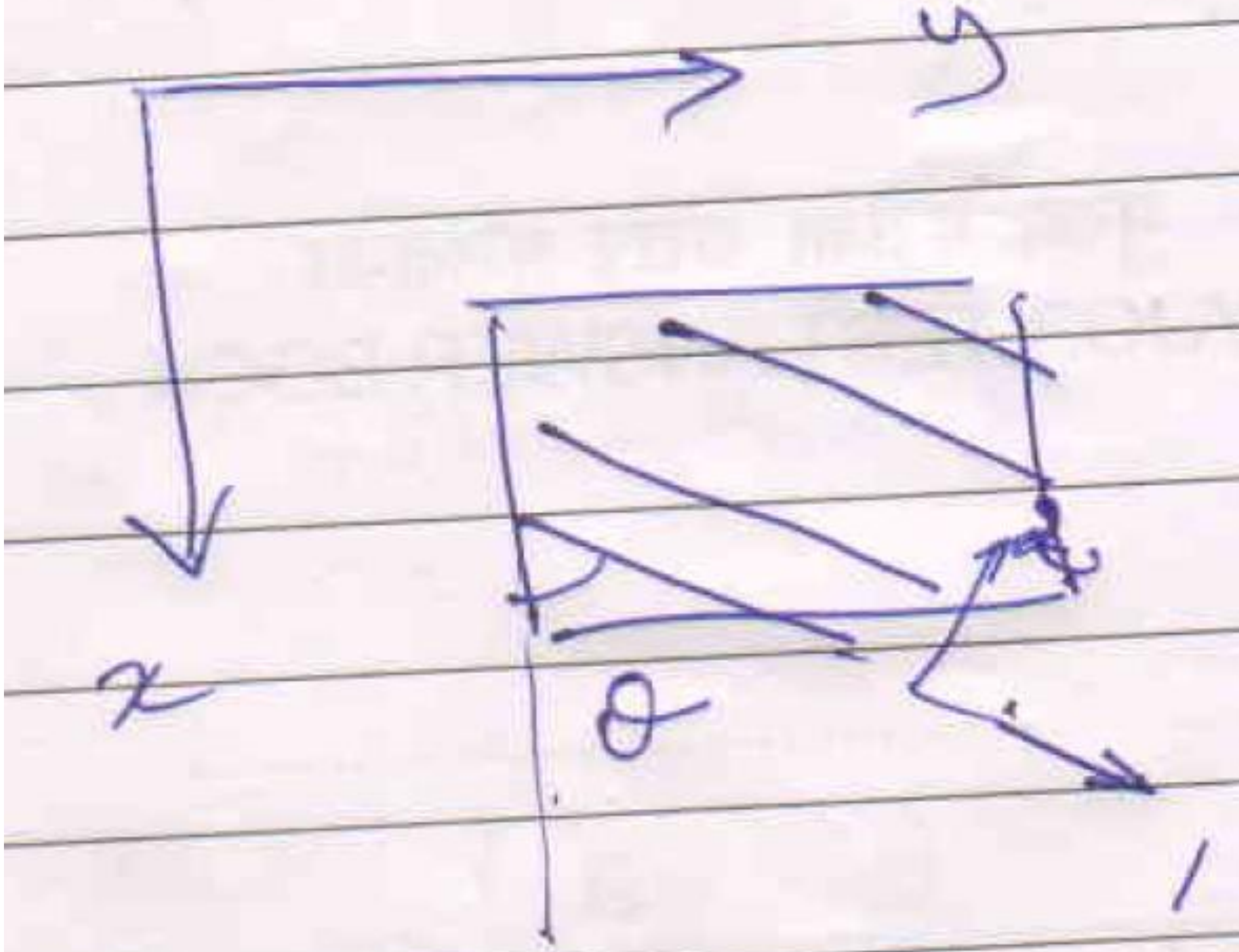
Composites and Thermal Loads


y
x
θ

For transversely isotropic

$$\frac{1}{G_{23}} = \frac{2(1+\nu_{23})}{E_2}$$

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}$$

$$Q_{12} = Q_{21} = \frac{\nu_{12} E_2}{1-\nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = T_1 \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

$$T_1 = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & -2\cos\theta\sin\theta \\ -\cos\theta\sin\theta & \sin\theta\cos\theta & \cos^2\theta-\sin^2\theta \end{bmatrix}$$

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} = T_2 \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$[T_2] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & -\cos\theta\sin\theta \\ -2\cos\theta\sin\theta & 2\cos\theta\sin\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T_1]^{-1} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} [T_2] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{xy} & Q_{yy} & Q_{ys} \\ Q_{xs} & Q_{ys} & Q_{ss} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

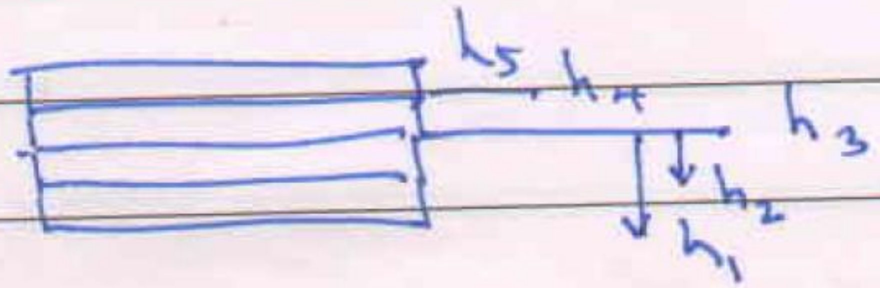
$$\begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{xy} & Q_{yy} & Q_{ys} \\ Q_{xs} & Q_{ys} & Q_{ss} \end{bmatrix} = [T_1]^{-1} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} [T_2]$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} z \, dz$$

$$= \int_{-h/2}^{h/2} [Q]_k \, z^2 \begin{Bmatrix} \dfrac{\partial \beta_x}{\partial x} \\ \dfrac{\partial \beta_y}{\partial y} \\ \dfrac{\partial \beta_x}{\partial y} + \dfrac{\partial \beta_y}{\partial x} \end{Bmatrix} dz$$

$$= [D] \begin{Bmatrix} \beta_{x,x} \\ \beta_{y,y} \\ \beta_{x,y} + \beta_{y,x} \end{Bmatrix}$$

$$D = \sum_{k=1}^{n} [Q]_k \frac{h_k^3 - h_{k-1}^3}{3}$$



$h_1 = -h/2$

$h_5 = h/2$

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} \cos\theta & +\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} \sigma_{23} \\ \sigma_{13} \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{bmatrix} \cos\theta & +\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} \gamma_{23} \\ \gamma_{13} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ +\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} C_{23} & 0 \\ 0 & C_{13} \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

I rate enthusiasm even above professional skill.

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} dz$$

$$= \sum_{k=1}^{n} \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}_k \begin{bmatrix} G_{13} & 0 \\ 0 & G_{23} \end{bmatrix}_k \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}_k \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

- So in the thickness direction Q for each layer is different. The B matrix is same for all layers.
- Once displacement has been calculated stress in a particular element in a layer is calculated using the Q matrix for that layer.

- Thermal Loads for Isotropic Materials

Thermal loads

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} - \alpha \Delta T \\ \epsilon_{yy} - \alpha \Delta T \\ \gamma_{xy} \end{Bmatrix}$$

$$M = \int \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} z \, dz$$

$$= \int \begin{bmatrix} & & \end{bmatrix} z \begin{Bmatrix} -z \frac{d\beta}{dx} - \alpha \Delta T \\ -z \frac{\partial \beta}{\partial y} - \alpha \Delta T \\ \gamma_{xy} \end{Bmatrix}$$

γ_{xy}

$$= \int \begin{bmatrix} & & \end{bmatrix} z \begin{Bmatrix} -z\dfrac{d\beta}{dx} \\ -z\dfrac{\partial\beta_y}{\partial y} \\ -z\left(\dfrac{\partial\beta_x}{\partial y} + \dfrac{\partial\beta_y}{\partial x}\right) \end{Bmatrix} dz$$

$$- \int \begin{bmatrix} & & \end{bmatrix}_{3\times3} z \begin{Bmatrix} -\alpha\Delta T \\ -\alpha\Delta T \\ 0 \end{Bmatrix}_{3\times1} dz$$

This term goes to RHS and contributes to the thermal load

$$V.W = \int \left[\frac{\partial \delta \beta_x}{\partial x} \quad \frac{\partial \delta \beta_y}{\partial y} \quad \frac{\partial \delta \beta_y}{\partial x} + \frac{\partial \delta \beta_x}{\partial y} \right]$$

$$\left[\quad \right] z \begin{Bmatrix} -\alpha \Delta T \\ -\alpha \Delta T \\ 0 \end{Bmatrix} dz$$

Thermal load

$$\int B_B^T \; z \left[\quad \right] \begin{Bmatrix} \alpha \Delta T \\ \alpha \Delta T \\ 0 \end{Bmatrix} dz$$

- For composites:

$$\alpha_x = m^2\alpha_1 + n^2\alpha_2$$

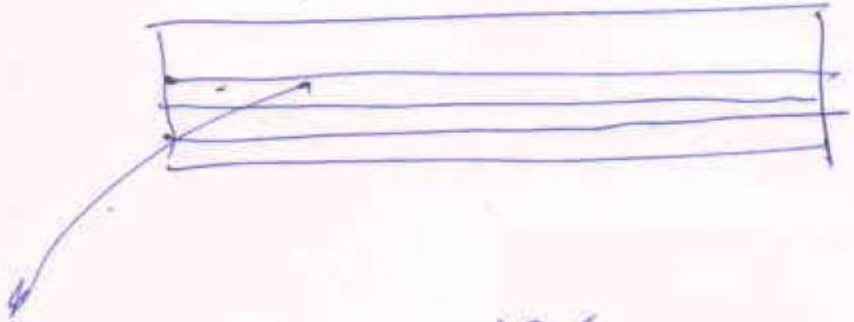
$$\alpha_y = n^2\alpha_1 + m^2\alpha_2$$

$$\gamma_{xy} = 2mn(\alpha_1 - \alpha_2)$$

$$\alpha_{xz} = 0$$

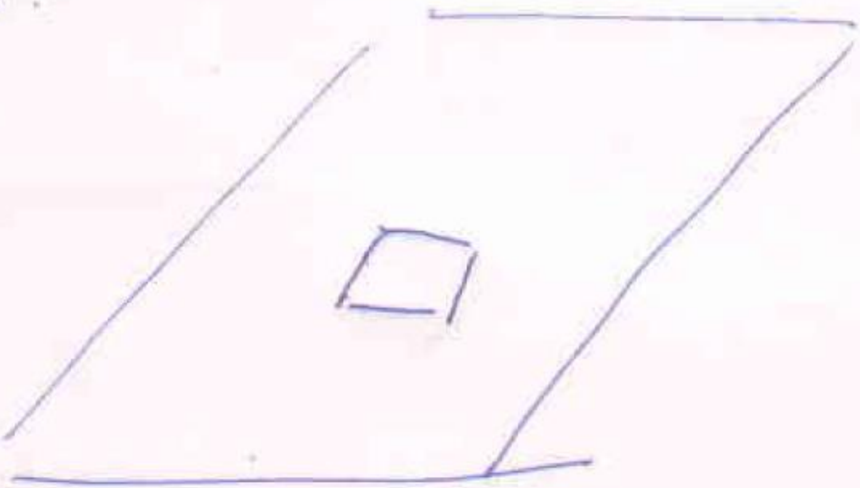
$$\alpha_{yz} = 0$$

m= cos θ, n=sin θ



K^{th} layer

yer



ement in K^{th} layer

$$\{\sigma\}_k^e = [Q]_k^e [B]^e \{q\}^e$$

$$\{q\}^e = \begin{Bmatrix} w_1 \\ \theta_{x1} \\ \theta_{y1} \\ \vdots \\ w_4 \\ \theta_{x4} \\ \theta_{y4} \end{Bmatrix}^e$$